Sheet (4) Solution
Kinematics of fluid flow (continuity equation)

Problem One

Find the acceleration and stream line function for the flows given below, then sketch the stream lines

(i) u=4, v=-3

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = 0$$
$$\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial v}{\partial y} = 0$$

$\therefore a_x = 4(0) - 3(0) = 0$

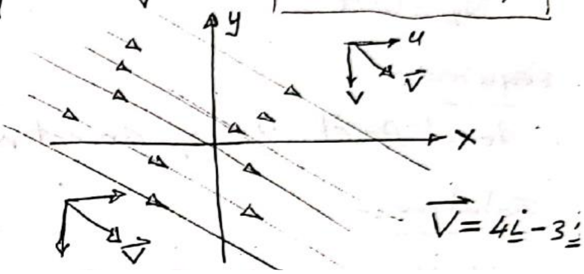
$\therefore a_y = 4(0) - 3(0) = 0$

To get stream func. $\frac{dx}{u} = \frac{dy}{v}$ OR $\boxed{v\,dx = u\,dy}$

$\therefore \int -3\,dx = \int 4\,dy$

$\therefore -3x = 4y + C'$

$\therefore y = \frac{-3}{4}x + C$ (line)



(ii) u=4, v=-3x

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = -3$$
$$\frac{\partial u}{\partial y} = 0 \qquad \frac{\partial v}{\partial y} = 0$$

$\therefore a_x = 4(0) - 3x(0) = 0$

$\therefore a_y = 4(-3) - 3x(0) = -12$

stream func.

$\int -3x\,dx = \int 4\,dy$

$-3\frac{x^2}{2} = 4y + C'$

$\therefore y = \frac{-3}{8}x^2 + C$ (Parabola)

(iii) u=4y, v=-3x

$\therefore a_x = 4y(0) - 3x(4) = -12x$

$\therefore a_y = 4y(-3) - 3x(0) = -12y$

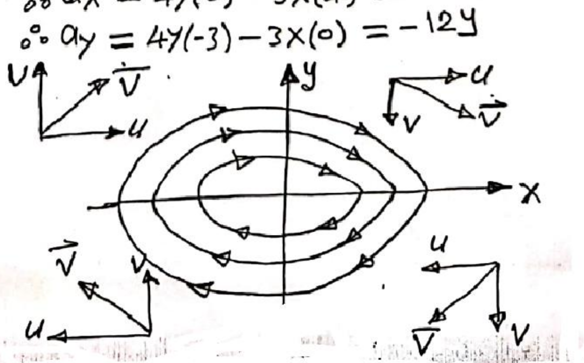
$$\therefore \frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = -3$$
$$\frac{\partial u}{\partial y} = 4 \qquad \frac{\partial v}{\partial y} = 0$$

stream $\int -3x\,dx = \int 4y\,dy$

$\therefore \frac{-3}{2}x^2 = \frac{4}{2}y^2 + C'$

$\therefore 3x^2 + 4y^2 = C$ (Ellipse)

$\therefore \frac{x^2}{C/3} + \frac{y^2}{C/4} = 1$



(iv) u=4y, v=3x

(iv) $u = 4y$ & $v = 3x$

$\therefore \frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = 3$

$\frac{\partial u}{\partial y} = 4 \qquad \frac{\partial v}{\partial y} = 0$

$\therefore a_x = 4y(0) + 3x(4) = 12x$

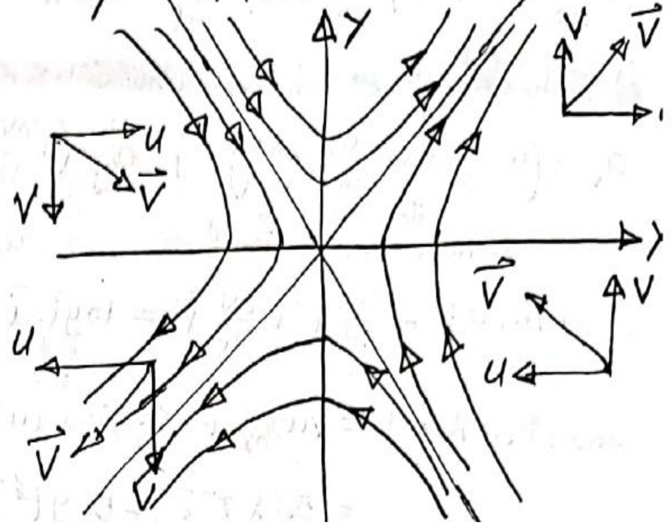
$\therefore a_y = 4y(3) + 3x(0) = 12y$

Stream

$$\int 3x\,dx = \int 4y\,dy$$

$$\therefore \frac{3x^2}{2} = \frac{4y^2}{2} + C$$

$$\therefore 3x^2 - 4y^2 = C \text{ (Hyperbola)}$$



(v) u=4y, v=-4x

(v) $u = 4y$ & $v = -4x$

$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial x} = -4$

$\frac{\partial u}{\partial y} = 4 \qquad \frac{\partial v}{\partial y} = 0$

$\therefore a_x = 4y(0) - 4x(4) = -16x$

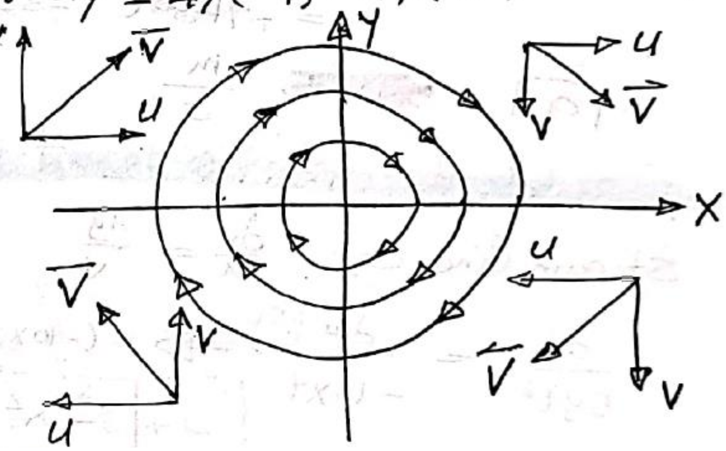
$\therefore a_y = 4y(-4) - 4x(0) = -16y$

stream:

$$\int -4x\,dx = \int 4y\,dy$$

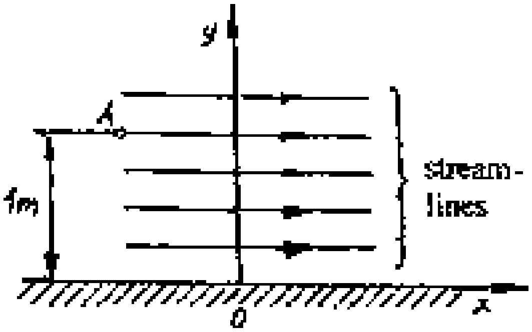
$$\frac{-4x^2}{2} = \frac{4y^2}{2} + C$$

$$\therefore x^2 + y^2 = C \text{ (circle)}$$



Problem two

Unsteady, two dimensional flow, $V_y=0$, $V_x=5yt^2$, Calculate the local and convective acceleration in point 'A' at t=0.5 s.



$V=V_y=0 \rightarrow a_y=0.$

$u=V_x=5yt^2 \rightarrow a_x=u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}+\frac{\partial u}{\partial t}$

$a_{x\,conv}=u\cancel{\frac{\partial u}{\partial x}}+v\cancel{\frac{\partial u}{\partial y}}=(5yt^2)(0)+(0)(5t^2)=zero$

$a_{x\,local}=\frac{\partial u}{\partial t}=5y(2t)=10yt$

$\left.\frac{\partial u}{\partial t}\right)_{at\,A}=10(1)(0.5)=5\ m/s^2$

from Fig. given.

Problem Three

If the velocity distribution in a circular pipe of radius R is given by u= $U_{max}\left(1-\frac{r^2}{R^2}\right)$ where r is the radial distance from center and U_{max} is the maximum flow velocity at the center, find the drag force on a section of pipe 20cm long , 12cm diameter. Use U_{max}=3m/s and μ=0.011 poise. Sketch the velocity distribution.

$u=U_{max}\left(1-\frac{r^2}{R^2}\right)$ drag force = shear force

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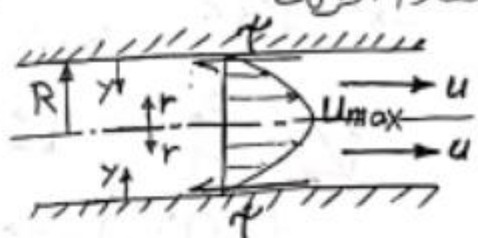


$\because \tau=\mu.\frac{du}{dy}=-\mu.\left.\frac{du}{dr}\right)_{r=R}$

$\therefore F_{shear}=\tau.A_{surface}(2\pi RL)$

$\because \left.\frac{du}{dr}\right)_R=U_{max}\left(0-\frac{2r}{R^2}\right)_R$

$=U_{max}\frac{-2R}{R^2}=\frac{-2U_{max}}{R}$

$\therefore F=-\mu\left(\frac{-2U_{max}}{R}\right)(2\pi RL)$

$F_D=4\pi(0.011*10^{-1})(3)(20)=0.829\text{N}$

Problem four

Unsteady, two dimensional flow, $u=5yt^2$, $v=-10xt$

(i) Calculate the local and convective acceleration at point A (-1,1) at t= 2 sec

(ii) Calculate the total acceleration at (-1,1) at t= 2 sec

(iii) Sketch the stream lines at t=2 sec & 1 sec

$$u = 5yt^2 = 5\,(1)(2)^2 = 20$$

$$\frac{\partial u}{\partial x} = \text{zero}$$

$$\frac{\partial u}{\partial y} = 5t^2 = 20$$

$$\frac{\partial u}{\partial t} = 10yt = 10 \times 1 \times 2 = 20$$

$$v = -10xt = (-10)(-1)(2) = 20$$

$$\frac{\partial v}{\partial x} = -10t = -20$$

$$\frac{\partial v}{\partial y} = \text{zero}$$

$$\frac{\partial v}{\partial t} = -10x = -10 \times -1 = 10$$

(i) Calculate the local and convective acceleration at point A (-1,1) at t= 2 sec

$$\bar{a} = a_x \bar{i} + a_y \bar{j}$$

$$a_x = \left\{ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right\} + \left\{ \frac{\partial u}{\partial t} \right\}$$

$$a_y = \left\{ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right\} + \left\{ \frac{\partial v}{\partial t} \right\}$$

Convective Accel. Local Accel.

$$\text{Local Accel.} = \frac{\partial u}{\partial t}\bar{i} + \frac{\partial v}{\partial t}\bar{j} = 20\bar{i} + 10\bar{j}$$

$$\text{Convective Accel.} = \left(\cancel{u\frac{\partial u}{\partial x}} + v\frac{\partial u}{\partial y}\right)\bar{i} + \left(u\frac{\partial v}{\partial x} + \cancel{v\frac{\partial v}{\partial y}}\right)\bar{j}$$

$$= 20 \times 20\,\bar{i} + 20 \times (-20)\,\bar{j}$$

$$= 400\,\bar{i} - 400\,\bar{j}$$

(ii) Calculate the total acceleration at (-1,1) at t= 2 sec

Total Acceleration = convective Accel. + Local Accel.

= 420 i – 390 j

$$|a| = 573.15\ m/s^2$$

(iii) Sketch the stream lines at t=2 sec & 1 sec

Stream line eq^{n} $\dfrac{dx}{u} = \dfrac{dy}{v}$

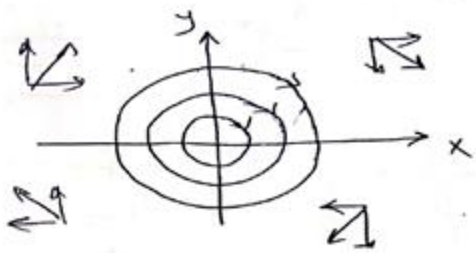
$$\frac{dx}{5yt^2} = \frac{dy}{-10xt} \Rightarrow (-\cancel{10}^{\,2}x\,\cancel{t})\,dx = (5yt^{2})\,dy$$

$$\boxed{-x^2 = \frac{y^2}{2}t + c}$$

at t = 2 sec

$-x^2 = y^2 + c \Rightarrow x^2 + y^2 = c$

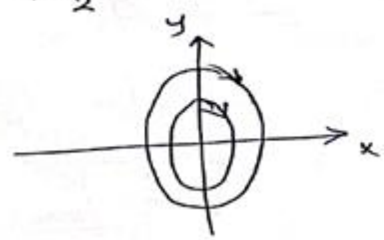
$\rightarrow$ Circle.



at t = 1 sec.

$-x^2 = \dfrac{y^2}{2} + c$

$\dfrac{x^2}{1} + \dfrac{y^2}{2} = c$



Problem Five

Find and sketch the stream line for the given below and calculate the acceleration at P (-1,2)

(i) u=x, v=-y

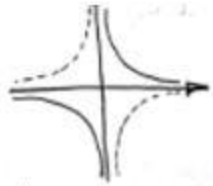
(ii) u=x, v=y

(iii) u=y², v=xy

(iv) u=y², v=-xy

(v) u=x, v=2y

(i) $u = x$, $v = -y$



$\therefore \int \frac{dx}{x} = \int \frac{dy}{-y}$

$\therefore \ln x + \ln y = c'$

$\therefore \ln xy = \ln c$

$\therefore xy = c$

$\therefore y = c/x$ (hyperbola)

(ii) $u = x$ $v = y$

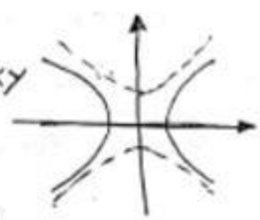
$\therefore \int \frac{dx}{x} = \int \frac{dy}{y}$

$\therefore \ln x + c = \ln y$

$\ln cx = \ln y$

$\therefore y = cx$ (line)

(iii) $u = y^2$ $v = xy$



$\frac{dx}{y^2} = \frac{dy}{xy}$

$\int x\,dx = \int y\,dy$

$\frac{x^2}{2} = \frac{y^2}{2} + c$

$x^2 - y^2 = c$ (hyperbola)

(iv) $u = y^2$ $v = -xy$

$\frac{dx}{y^2} = \frac{dy}{-xy}$

$\int x\,dx = \int -y\,dy$

$\frac{x^2}{2} + \frac{y^2}{2} = c$

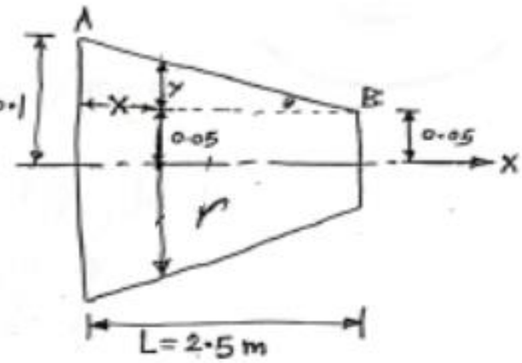
$x^2 + y^2 = c$ circle

Problem Six

A conical nozzle 2.5m long converging from 0.2m to 0.1m diameter linearly is subjected to a constant flow rate of 1m $^3/s$. Determine the acceleration at the mid-length of the nozzle. Assume uniform flow over each cross section.

$Q = 1\ m^3/sec$

$a)_{x=\frac{L}{2}} = ??$ $\quad u)_{x=\frac{L}{2}} = ??$



$\boxed{\therefore\ a_x = u \cdot \frac{du}{dx}}$ one dim

from ABc $\quad \frac{y}{0.05} = \frac{2.5 - x}{2.5}$

$\therefore y = 0.05 - 0.02\,x$

$\therefore r = (y + 0.05) = 0.1 - 0.02\,x \quad \Rightarrow \quad r = 0.075\,m$

$\therefore A = 0.01767$

$\therefore U_x = \frac{Q}{A_x} = \frac{1}{\pi(0.1 - 0.02x)^2}$

$U_A = 31.83\ m/s$

$U_B = 127.32\ m/s$

$\therefore U_x = \frac{1}{\pi}(0.1 - 0.02\,x)^{-2}$

$\therefore \frac{du}{dx} = \frac{-2}{\pi}(0.1 - 0.02\,x)^{-3}(-0.02)$

at $x = 1.25$

$\therefore u = \frac{1}{\pi}\left[0.1 - 0.02(1.25)\right]^{-2} = 56.59\ m/s$ #

$\therefore \frac{du}{dx} = \frac{0.04}{\pi}\left(0.1 - 0.02(1.25)\right)^{-3} = 30.18\ 1/s$

$\therefore a_x = (56.59)(30.18) = 1707.9\ m/s^2$ #

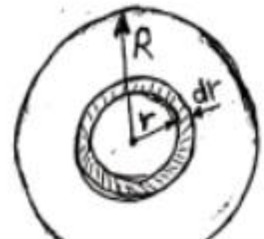
$a_x = u \cdot \frac{du}{dx} = \frac{0.04}{\pi^2(0.1 - 0.02x)^5}$

Problem Seven:

For a laminar flow in a tube where the velocity distribution is given by $U = Umax\left(1 - \frac{r^2}{R^2}\right)$ where **R** is the tube radius and **Umax** is the center line . Show that the mean velocity is half the center line velocity.

$$U = U_{max}(1 - r^2/R^2)$$

$$\because U_{\substack{mean\\avrg}} = \frac{\int u \cdot dA}{A}$$



$$\therefore U_{\substack{mean\\avrg}} = \int_0^R U_{max}(1 - r^2/R^2) \cdot 2\pi r\, dr / \pi R^2$$

$$= \frac{2\pi U_{max.}}{\pi R^2} \int_0^R (r - r^3/R^2)\, dr$$

$$= \frac{2\, U_{max}}{R^2}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)_0^R$$

$$= \frac{2\, U_{max}}{R^2}\left(\frac{R^2}{2} - \frac{R^4}{4R^2}\right) = \frac{U_{max}}{2}$$

Problem Eight:

The velocity vector of a fluid particle is given by

$u = x^2y + t^2,\ \ v = x^2 + y^2 + z^2 + 3t^4\ ,\ \ w = x^3 + z^2 + t.$

Calculate the velocity and acceleration of the particle at point **(2,3,1)** after **2 sec**

$P(\overset{x}{2}, \overset{y}{3}, \overset{z}{1}) \qquad t = 2\ sec$

$U = x^2 y + t^2 \qquad V = x^2 + y^2 + z^2 + 3t^4 \qquad W = x^3 + z^2 + t$

$\frac{\partial u}{\partial x} = 2xy = 12 \qquad \frac{\partial v}{\partial x} = 2x = 4 \qquad \frac{\partial w}{\partial x} = 3x^2 = 12$

$\frac{\partial u}{\partial y} = x^2 = 4 \qquad \frac{\partial v}{\partial y} = 2y = 6 \qquad \frac{\partial w}{\partial y} = 0$

$\frac{\partial u}{\partial z} = 0 \qquad \frac{\partial v}{\partial z} = 2z = 2 \qquad \frac{\partial w}{\partial z} = 2z = 2$

$\frac{\partial u}{\partial t} = 2t = 4 \qquad \frac{\partial v}{\partial t} = 12t^3 = 96 \qquad \frac{\partial w}{\partial t} = 1$

To evaluate velocity

$u = (2)^2(3) + (2)^2 = 16$

$v = (2)^2 + (3)^2(1)^2 + 3(2)^4 = 62$

$w = (2)^3 + (1)^2 + 2 = 11$

$\therefore \bar{V} = 16\underline{i} + 62\underline{j} + 11\underline{k}$

$\therefore |\bar{V}| = \sqrt{(16)^2 + (62)^2 + (11)^2} = 64.97 \ m/s$

$\because a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} = 444 \ m/s^2$

$\because a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} = 554 \ m/s^2$

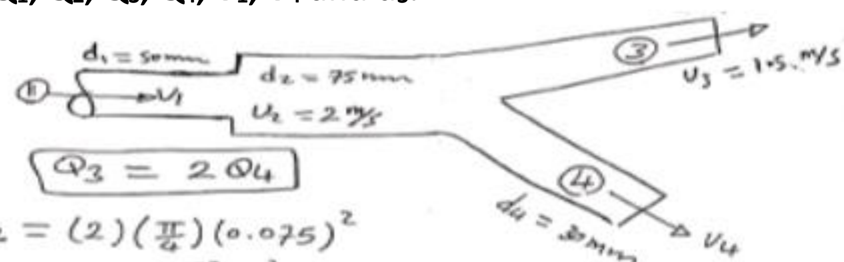
$\because a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} = 215 \ m/s^2$

$\therefore \bar{a} = 444\underline{i} + 554\underline{j} + 215\underline{k}$

$\therefore |\bar{a}| = \sqrt{(444)^2 + (554)^2 + (215)^2} = 741.8 \ m/s^2$

Problem Nine

Water flows through the shown pipe system. Such that the discharge Q_2 divides so that $Q_3=2Q_4$. Calculate the values Q_1, Q_2, Q_3, Q_4, V_1, V_4 and d_3.



$\boxed{Q_3 = 2Q_4}$

$Q_1 = Q_2 = (2)(\frac{\pi}{4})(0.075)^2$

$Q_1 = Q_2 = 8.84 \times 10^{-3} \ m^3/s$ #

$\therefore V_1 = \frac{8.84 \times 10^{-3}}{\frac{\pi}{4}(0.050)^2} = 4.5 \ m/s$ #

$\because Q_2 = \cancel{Q_3}^{2Q_4} + Q_4$

$\therefore 8.84 \times 10^{-3} = 3Q_4 \longrightarrow Q_4 = 2.95 \times 10^{-3} \ m^3/s$

$\therefore Q_3 = 5.9 \times 10^{-3} \ m^3/s$

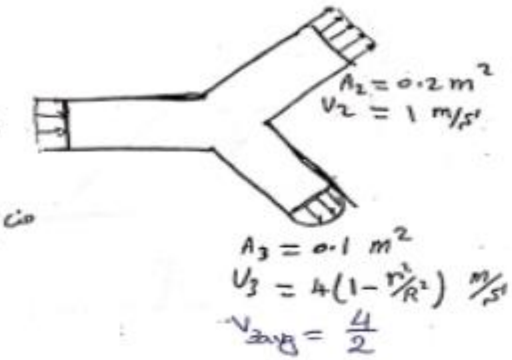
$\because Q_3 = \frac{\pi}{4} d_3^2 \cdot V_3$

$\therefore 5.9 \times 10^{-3} = \frac{\pi}{4}(1.5)d_3^2 \longrightarrow d_3 = 70.8 \ mm$

$\therefore U_4 = \frac{2.95 \times 10^{-3}}{\frac{\pi}{4}(0.030)^2} \Longrightarrow U_4 = 4.17 \ m/s$

Problem Ten

An incompressible fluid flows steadily through a duct which has two outlets. The flow is one-dimension at section (1) (2), but the velocity profile is Parabolic at section (3). What is the velocity V_1?



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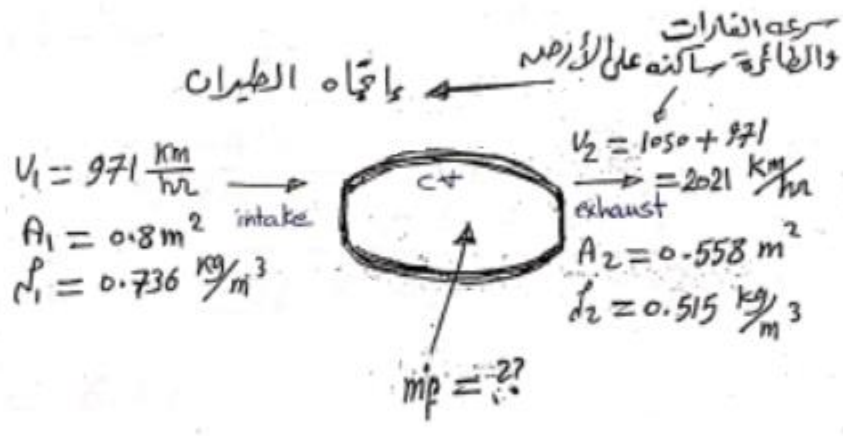
$$V_3 = \frac{4}{2} = 2 \quad m/s$$

$$\because \ V_1 A_1 = V_2 A_2 + V_3 A_3$$

$$\therefore \ V_1 (0.3) = (1)(0.2) + (2)(0.1) \qquad \therefore V_1 = 4/3 \ m/s$$

Problem Eleven

An airplane moves forward at a speed of 971 Km/hr. The frontal intake area of the jet engine is 0.8m² and the entering air density is 0.736 Kg/m³. A stationary observer estimates that relative to the earth, the jet engine exhaust gases moves away from the engine with speed of 1050 Km/hr. The engine exhaust area is 0.558m² and the exhaust gas density is 0.515 Kg/m³. Estimate the mass flow rate of the fuel into the engine in Kg/hr.



$$\because \ \rho_1 V_1 A_1 + \dot{m}_f = \rho_2 V_2 A_2$$

$$\therefore \ (0.736)(971 \times 10^3)(0.8) + \dot{m}_f = (0.515)(2021 \times 10^3)(0.558)$$

$$\text{so } \dot{m}_f \cong 9050 \ Kg/hr$$

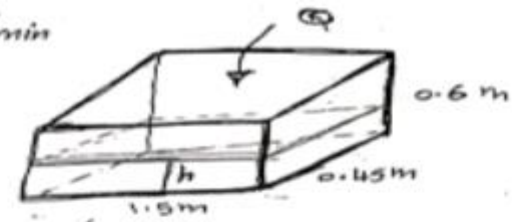
Problem Twelve

A bathtub is being filled with water from a faucet. The rate of flow, Q From the faucet is steady at 0.0045m³/min. The tub volume is approximated by a rectangular space. Determine the rate of change of the depth of water in the tub in m/min at any instant

$Q = 0.0045 \ m^3/min$

$\dot{m} = 4.5 \ kg/min$

Continuity equ



$$\therefore \iint_S \rho \, \bar{u} \cdot d\bar{A} + \frac{\partial}{\partial t} \iiint_V \rho \, dV = 0$$

$$\therefore \underbrace{-0.0045}_{\text{الداخل}} + \underbrace{0}_{\text{الخارج}} + \frac{\partial}{\partial t} \left[h \, (1.5 \times 0.45) \right] = 0$$

$$\therefore \frac{\partial h}{\partial t} = \frac{0.0045}{0.675} = 6.67 \times 10^{-3} \ m/min = 6.67 \ mm/min$$

حساب الزمن اللازم لملئ الخزان

$$\therefore \frac{dh}{dt} = 6.67 \times 10^{-3} \ m/min$$

$$\therefore \int_0^{0.6} dh = \int_0^{T} 6.67 \times 10^{-3} \, dt$$

$$\therefore (0.6 - 0) = 6.67 \times 10^{-3} \, T$$

$$\therefore T \cong 90 \ min = 1.5 \ hr$$

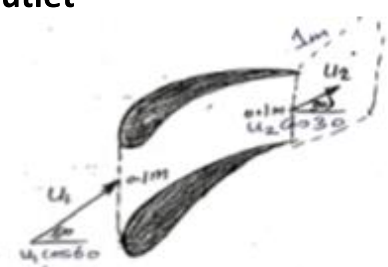
Problem Thirteen

An incompressible fluid flows steadily between a pair of vanes as shown in figure, the average velocity at entrance to the vane is 10m/s .Determine the volumetric flow rate per unit depth between the vanes and the average velocity at outlet

$u_1 = 10 \ m/s$

$Q = ??$

$u_2 = ??$



sol:

$$u_1 \cos 60 \, (\cancel{A_1}) = u_2 \cos 30 \, (\cancel{A_2})$$

$$\therefore u_2 = 10 \cos 60 / \cos 30 = 5.77 \ m/s$$

$$\therefore Q = (10 \cos 60)(0.1 * 1) = 0.5 \ m^3/sec/m \text{ wide}$$

Problem Fourteen

Oil (S=0.91) enters at section (1) in the shown figure at a weight flow rate of 250N/hr to lubricate the thrust bearing. Oil exits radial through the narrow clearance between thrust plates. Calculate the outlet volume flow rate and the outlet average velocity.

$$\dot{m}_1 = \frac{250 = \dot{w}}{g = 9.81} = \dot{m}_2$$

$$= 25.48 \text{ Kg/h}$$

$$\therefore Q_2 = \frac{25.48}{910} = \frac{\dot{m}}{\rho}$$

$$Q_2 = 0.028 \text{ m}^3/\text{hr}$$

Thickness

$$\therefore Q_2 = V_2 \, (\pi D) \cdot t \quad A_2$$

$$\therefore 0.028 = V_2 \left(\pi (0.10)(0.002)\right)$$

$$\therefore V_2 = 44.56 \text{ m/hr}$$

$$= 1.24 \text{ cm/sec}$$