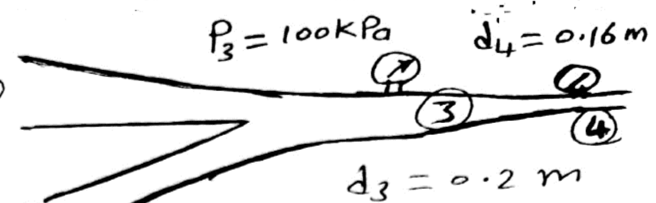
Sheet 5

(1) Water and alcohol are mixed in the y-duct shown in figure. What is the average density and average velocity? If the 0.2m diameter pipe tapers to 0.16m diameter and the losses encountered are $\frac{3v^2}{2g}$, where v is the velocity at pipe inlet, find the final pressure

$d_1 = 0.1\ m$
$V_1 = 3\ m/s$
$\rho_1 = 1000\ kg/m^3$

$d_2 = 0.1 m$
$V_2 = 2 m/s$
$\rho_2 = 800\ kg/m^3$



Req:

$\rho_3 = ??$
$V_3 = ??$
$P_4 = ??$

$$\underset{3-4}{\text{losses}} = 3\frac{V_3^2}{2g}$$

$$\boxed{\dot{V} = Q}$$

incompressible للسوائل فقط

$\therefore\ \dot{m}_3 = \rho_3 \cdot \dot{V}_3$

Sol: $\therefore\ \dot{m}_1 + \dot{m}_2 = \dot{m}_3$ & $\boxed{\dot{V}_1 + \dot{V}_2 = \dot{V}_3}$

$\therefore\ \dot{m}_3 = \rho_1 V_1 \cdot A_1 + \rho_2 V_2 A_2$ & $\dot{V}_3 = V_1 A_1 + V_2 A_2$

$\therefore\ \dot{m}_3 = (10^3)(3)\frac{\pi}{4}(0.1)^2 + (800)(2)\frac{\pi}{4}(0.1)^2 = 36.128\ kg/sec$

$\therefore\ \dot{V}_3 = (3)\frac{\pi}{4}(0.1)^2 + (2)(\frac{\pi}{4})(0.1)^2 = 0.03927\ m^3/sec$

$\therefore\ \rho_3 = \frac{m_3}{\dot{V}_3} \frac{36.128}{0.03927} \simeq 920\ kg/m^3$

$\therefore\ V_3 = \frac{\dot{V}_3}{A_3} = \frac{0.03927}{\frac{\pi}{4}(0.2)^2} = 1.25\ m/s$

Bernoulli's eq: between ④ ③

$\therefore\ V_3 A_3 = V_4 A_4 \longrightarrow V_4 = 1.25\left(\frac{0.2}{0.16}\right)^2 = 1.95\ m/s$

$$\frac{V_3^2}{2\cancel{g}} + \frac{P_3}{\cancel{\rho_3 g}} = \frac{V_4^2}{2\cancel{g}} + \frac{P_4}{\cancel{\rho_3 g}} + \boxed{\frac{3V_3^2}{2\cancel{g}}} \text{ Losses}$$

$$\therefore\ \frac{(1.25)^2}{2} + \frac{10^5}{920} = \frac{(1.95)^2}{2} + \frac{P_4}{920} + \frac{3(1.25)^2}{2}$$

$$\boxed{\begin{aligned} \therefore\ P_4 &= 96813\ Pa \\ &\approx 96.813 KPa \end{aligned}}$$

(2) For the siphon shown in Figure. Find the volume flow rate and the pressure at A, B. Sketch the T.E.L.

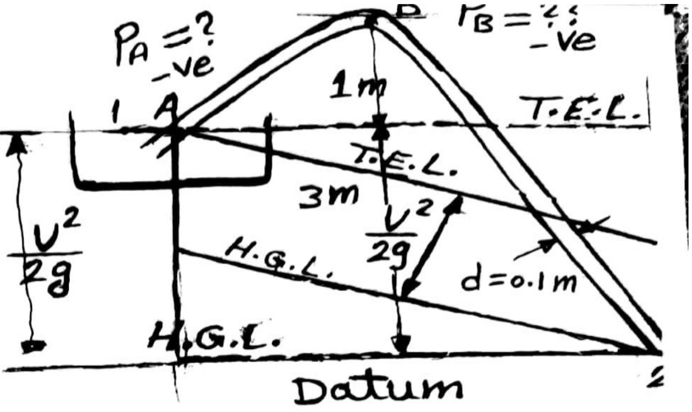
[If the loss in the pipe is $\frac{3v^2}{2g}$ .Sketch the T.E.L. and H.G.L.]

$\boxed{2}$ (i) $\dot{V} = Q = ??$

$P_A = ??$ $P_B = ??$

T.E.L. & H.G.L.

(ii) losses $= \frac{3V^2}{2g}$



sol: $\boxed{\text{(i) No losses}}$ Bernoulli (1)(2)

$$\underset{=0}{\cancel{\frac{V_1^2}{2g}}} + \underset{=0}{\cancel{\frac{P_1}{\rho g}}} + \underset{3}{\cancel{Z_1}} = \frac{V_2^2}{2g} + \underset{=0}{\cancel{\frac{P_2}{\rho g}}} + \underset{=0}{\cancel{Z_2}}$$

$\therefore V_2 = \sqrt{2g(3)} = 7.67\ m/s = V_A = V_B$

$\therefore Q = \dot{V} = V\cdot\frac{\pi}{4}d^2 = (7.67)\frac{\pi}{4}(0.1)^2 = 0.060\ m^3/s$

60 Litre/sec

Bernoulli (A) ②

$$\left(\cancel{\frac{V^2}{2g}} + \frac{P}{\rho g} + Z\right)_A = \left(\cancel{\frac{V^2}{2g}} + \underset{=0}{\cancel{\frac{P}{\rho g}}} + Z\right)_2$$

$\therefore \frac{P_A}{(1000)(9.81)} + 3 = 0 \longrightarrow \boxed{\therefore P_A = -29.43\ KPa}$

Bernoulli ① (B)

$$\left(\underset{=0}{\cancel{\frac{V^2}{2g}}} + \underset{=0}{\cancel{\frac{P}{\rho g}}} + \underset{3}{\cancel{Z}}\right)_1 = \left(\frac{V^2}{2g} + \frac{P}{\rho g} + \underset{4}{\cancel{Z}}\right)_B$$

$\therefore \cancel{3} = \cancel{\frac{(7.67)^2}{2g}} + \frac{P_B}{(1000)g} + 4$ $\boxed{\therefore P_B = -39.22 KPa}$

$\boxed{\text{(ii) With losses:}}$ Bernoulli ①②

$$\left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_1 = \left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_2 + h_{L_{1-2}}$$

$0 + 0 + 3 = \frac{V_2^2}{2g} + 0 + 0 + \frac{3V_2^2}{2g}$

$\therefore Q = V * \frac{\pi}{4} d^2$

$\therefore V_2 = 3.836\ m/s$

$\therefore Q = 0.030\ m^3/$

30 Litre/se

Bernoulli Ⓐ ②:

$$\left(\cancel{\frac{V^2}{2g}} + \frac{P}{\rho g} + Z\right)_A = \left(\cancel{\frac{V^2}{2g}} + \underset{=0}{\cancel{\frac{P}{\rho g}}} + \underset{=0}{\cancel{Z}}\right)_2 + \overset{\frac{3V^2}{2g}}{\cancel{h_{L_{A2}}}}$$

$$\therefore \frac{P_A}{(1000)g} + 3 = 0 + \frac{3(3.836)^2}{2g}$$

$$\boxed{\therefore P_A = -7.358\ \text{KPa}}$$

Also:

Bernoulli ① Ⓐ:

$$0 + 0 + 3 = \frac{(3.836)^2}{2g} + \frac{P_A}{(1000)g} \qquad \boxed{\therefore P_A = -7.357\ \text{K}}$$

Bernoulli ① Ⓑ:

$$h_{L_{1-B}} \overset{\text{نفرض}}{=} \frac{1}{3} h_{L_{1-2}} = \frac{1}{3}\left(\frac{3V^2}{2g}\right) = \frac{V^2}{2g}$$

$$\left(\underset{=0}{\cancel{\frac{V^2}{2g}}} + \underset{=0}{\cancel{\frac{P}{\rho g}}} + \underset{3}{\cancel{Z}}\right)_1 = \left(\frac{V^2}{2g} + \frac{P}{\rho g} + \underset{4}{\cancel{Z}}\right)_B + h_{L_{1-B}}$$

$$\therefore 3 = \cancel{\frac{(3.836)^2}{2g}} + \frac{P_B}{(1000)g} + 4 + \cancel{\frac{(3.836)^2}{2g}}$$

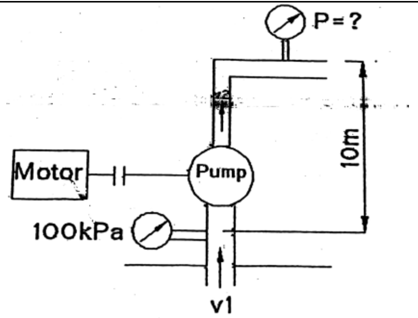
$$\boxed{\therefore P_B = -9.81\ \text{KPa}}$$

لاحظ: أن P_A ، P_B تتأثر بالسرعه داخل الأنبوبه

كلما زادت السرعه قل الضغط

Problem (3)

The water pump shown in figure is powered by a 15KW motor with an efficiency of 90%. If the flow rate is 0.3 m^3/min, inlet pipe is 20cm diameter and outlet pipe is 15cm diameter, find the gage reading , given the pressure at inlet is 100KPa, and the loss is $\frac{3v_2^2}{2g}$



3] $\text{Power} = Q \cdot \rho \cdot g \cdot h_P$

$$\boxed{\begin{array}{l} Q = 0.3/60 = 5*10^{-3}\ m^3/sec \\ Q = V_1A_1 = V_2A_2 \end{array}}$$

$\text{Power of pump} = 15000 * 0.9 = 13500$ W

$$\overset{\rho}{(1000)}\overset{Q}{\left(\frac{0.3}{60}\right)}\overset{g}{(9.81)}\ Hp = 13500$$

$\therefore Hp = 275.23$ m given: $h_L = \frac{3V_2^2}{2g}$

$$\therefore V_1 = \frac{Q}{A_1} = \frac{5*10^{-3}}{\frac{\pi}{4}(0.2)^2} = 0.159 \ m/sec$$

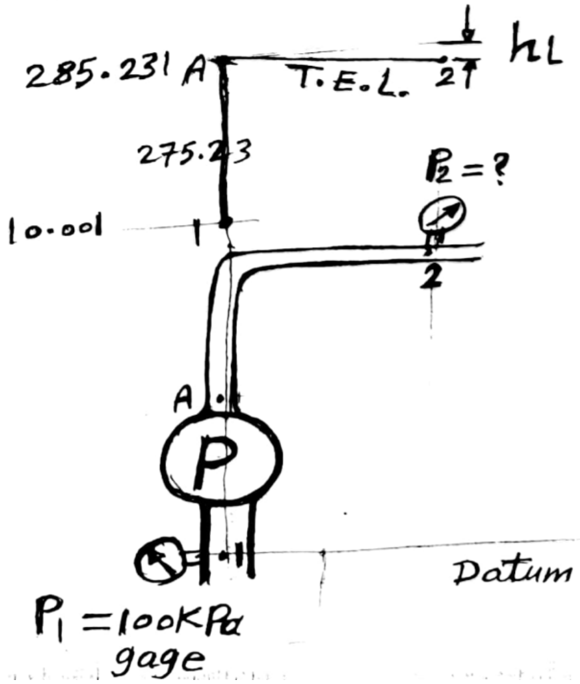
$$\therefore V_2 = \frac{5*10^{-3}}{\frac{\pi}{4}(0.15)^2} = 0.283 \ m/sec$$

Bernoulli ①②: $P_2 = ??$

$$\left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_1 + Hp = \left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_2 + h_{L_{1-2}}$$

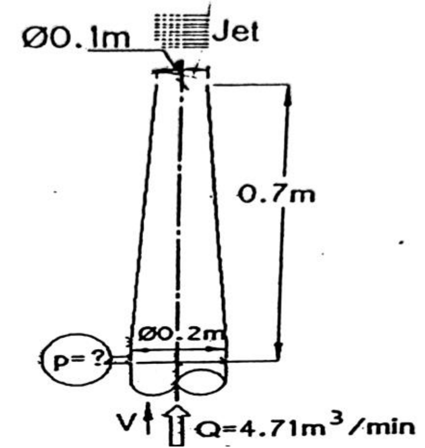
$$\frac{(0.159)^2}{2g} + \frac{10^5}{10^3\ g} + 0 + 275.23 = \frac{(0.283)^2}{2g} + \frac{P_2}{10^3 g} + 10 + \frac{3(0.28}{2g}$$

$$\boxed{\therefore P_2 = 2701.8 \ KPa} \simeq 2.7 \ MPa$$



Problem (4)

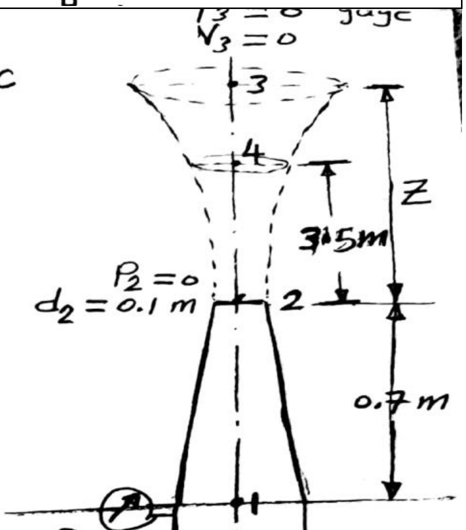
Find The gage reading, p, shown in figure, when water is flowing at the rate of 4.71 m^3/min, if the losses in the nozzle are $\frac{v^2}{4g}$ and v is the inlet velocity. Find the maximum height of the jet.



$\boxed{4}$ $Q = 4.71/60 = 0.0785 \ m^3/sec$

$\therefore V_1 = \frac{0.0785}{\frac{\pi}{4}(0.2)^2} = 2.50 \ m/s$

$\therefore V_2 = \frac{0.0785}{\frac{\pi}{4}(0.1)^2} = 10 \ m/s$



$P_1 = ?$

$d_1 = 0.2m$

$h_L = \frac{V_1^2}{4g}$

$Q = 4.71 \ m^3/min$

Bernoulli ①②:

$$\left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_1 = \left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_2 + h_{L_{1-2}}$$

$$\therefore \frac{(2.5)^2}{2g} + \frac{P_1}{10^3 * g} + 0 = \frac{(10)^2}{2g} + 0 + 0.7 + \frac{(2.5)^2}{4g}$$

$$\boxed{\therefore P_1 = 55.30 \ KPa}$$

Bernoulli ②③:

$$\frac{(10)^2}{2g} + 0 + 0 = 0 + 0 + Z$$

$$\boxed{\therefore Z = 5.097 \ m}$$

Bernoulli ②④: $\left(\frac{V^2}{2g} + \frac{P}{\gamma} + Z\right)_2 = \left(\frac{V^2}{2g} + \frac{P}{\gamma} + Z\right)_4$

$$\therefore \frac{(10)^2}{2g} + 0 + 0 = \frac{V_4^2}{2g} + 0 + 3.5$$

$\therefore V_4 = 5.6 \ m/s$ & $Q = 0.0785$

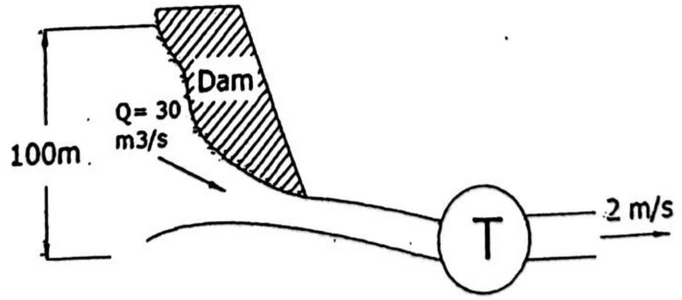
$\therefore Q = V_4 \cdot \frac{\pi}{4} d_4^2$

$\therefore 0.0785 = (5.6) \frac{\pi}{4} d_4^2$

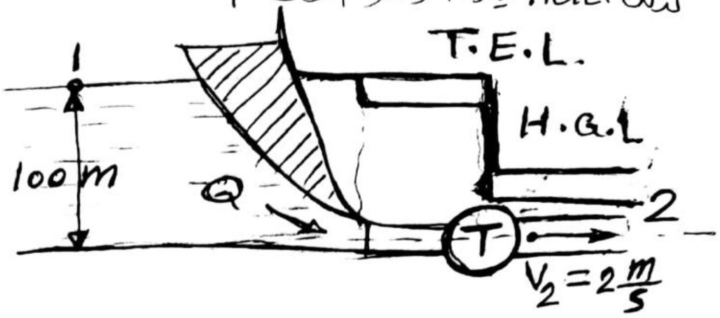
$$\boxed{\therefore d_4 = 0.134 m}$$

Problem (5)

A hydraulic power plant operates under the conditions shown in figure. Find the output power of the turbine, if the turbine efficiency is 95%.



غالبا الضغط [illegible] بعد [illegible]
لذلك .HGL يكون مرسوم تحت T



$\boxed{5}$ $Q = 30\ m^3/sec$

Power of turbine = ??

$\eta_T = 95\%$

Sol:

Power of Turbine $= \rho \cdot g \cdot H_T \cdot Q \cdot \eta_T$

Bernoulli ①②:

$$\left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_1 - H_T = \left(\frac{V^2}{2g} + \frac{P}{\rho g} + Z\right)_2 + h_{L_{1-2}}$$

$$\therefore\ 0 + 0 + 100 - H_T = \frac{(2)^2}{2g} + 0 + 0 + 0$$

$$\therefore\ H_T = 99.8\ m$$

$$\therefore\ Power = \overset{\rho}{(10^3)}\overset{g}{(9.81)}\overset{H_T}{(99.8)}\overset{Q}{(30)}\overset{\eta}{(0.95)}$$

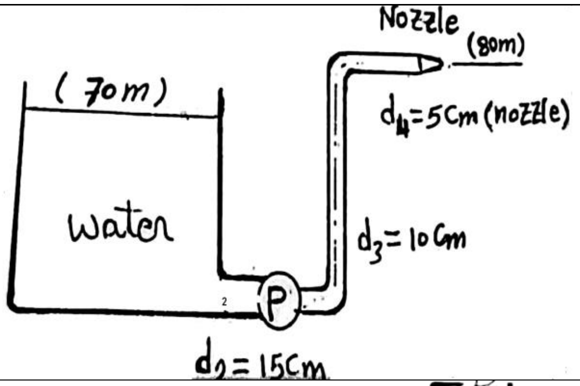
$$\boxed{Power = 27.9\ MW}$$

Notice:

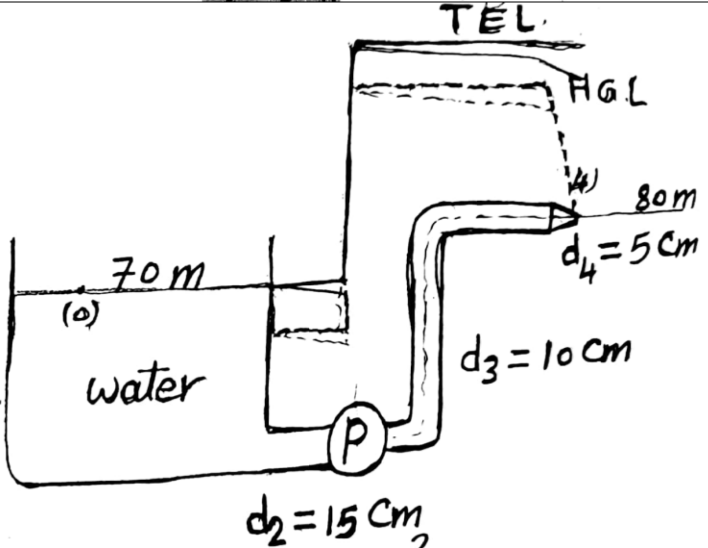
$$Fluid\ Power = \rho \cdot g \cdot Q \cdot H_T$$
$$= (9810)(30)(99.8)$$
$$= 29.37\ MW$$

Problem (6)

A pipe line with a pump leads to a nozzle as shown. Find the flow rate when the pump head is 40m assuming that the head losses are **h2**=$5v_2^2/2g$, **h3**=$12v_3^2/2g$, **h4**=$2v_4^2/2g$
then sketch the T.E.L and H.G.L.



$\boxed{\cancel{4}}$ ⑥ $H_p = 40\,m$



$\therefore Q = V_2 A_2 = V_3 A_3 = V_4 A_4$

$\therefore V_2 = \frac{1}{9} V_4 \qquad \therefore V_3 = \frac{1}{4} V_4$

$\therefore h_2 = 0.062 \frac{V_4^2}{2g}$

$\therefore h_3 = 0.75\, V_4^2/2g$

$\therefore h_4 = 2 \frac{V_4^2}{2g}$

$\therefore h_{Loss} = 2.812 \frac{V_4^2}{2g}$

Bernoulli (0)(4) :

$$\frac{P_o}{\gamma} + \frac{V_o^2}{2g} + Z_o + Hp = \frac{P_4}{\gamma} + \frac{V_4^2}{2g} + Z_4 + h_{losses}$$

$$0 + 0 + 70 + 40 = 0 + \frac{V_4^2}{2g} + 80 + 2.812 \frac{V_4^2}{2g}$$

$\boxed{\therefore V_4 = 12.426\, m/s}$

$\therefore Q = V_4 A_4 = 0.0244\, m^3/s$
$Q = 24.4$ Litre/s

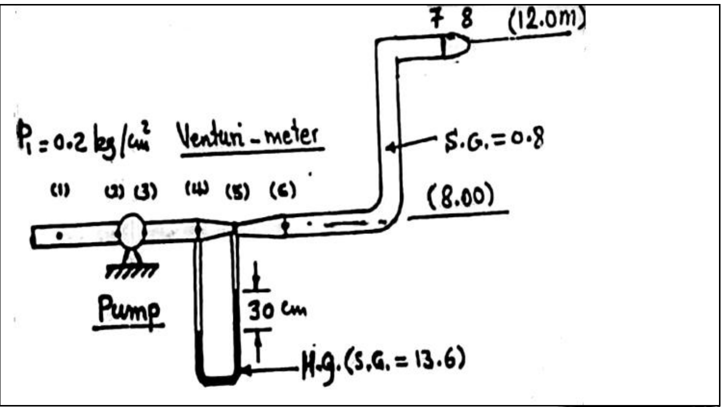
Problem (7)

For the shown pump, venture-meter and nozzle system, if $d_1 = d_2 = d_3 = d_4 = d_6 = d_7 = 20cm$, $d_5 = d_8 = 10cm$, $p_1 = 0.2$ bar,

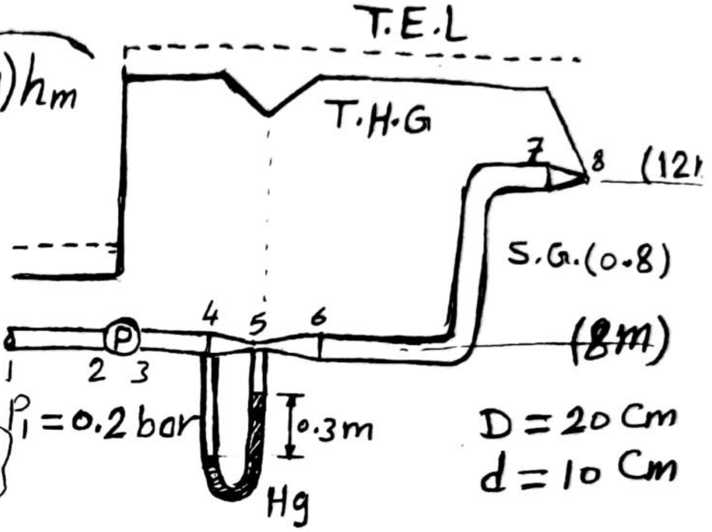
Find: (a) The discharge in (**m³/sec**).

(b) The pump power in (kW).

(c) Sketch the **T.E.L.** and the **H.G.L.**



(7) B Venturimeter:



$$\therefore Q = \frac{A_4 A_5}{\sqrt{A_4^2 - A_5^2}} \sqrt{2g\left(\frac{\rho_m}{\rho} - 1\right)h_m}$$

$$A_1 = A_4 = \frac{\pi}{4}(0.2)^2 = 0.03142\ m^2$$

$$A_8 = A_5 = \frac{\pi}{4}(0.1)^2 = 0.00785\ m^2$$

$$\boxed{\therefore Q = 0.0787\ m^3/s}$$

$$\boxed{\therefore V_8 = V_5 = \frac{0.0787}{0.00785} = 10\ m/s}$$

$$\boxed{\therefore V_1 = 2.5\ m/s}$$

Bernoulli (1)(8):

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 + H_p = \frac{P_8}{\gamma} + \frac{V_8^2}{2g} + Z_8$$

$$\frac{0.2 \times 10^5}{(9.81)(800)} + \frac{(2.5)^2}{2(9.81)} + 8 + H_p = 0 + \frac{(10)^2}{2(9.81)} + 12$$

$$\boxed{\therefore H_p = 6.23\ m}\ \#$$

$$\therefore Power = \rho . g . Q . H_p$$

$$= (800)(9.81)(6.23)(0.0787)$$

$$\boxed{Power = 3.848\ KW}\ \#$$

Problem (8)

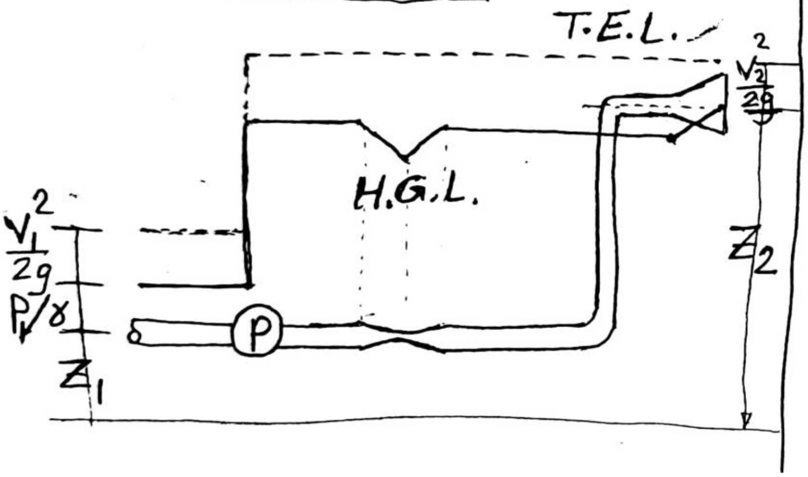
Sketch only the **T.E.L.** and the **H.G.L.** for the previous problem if we:

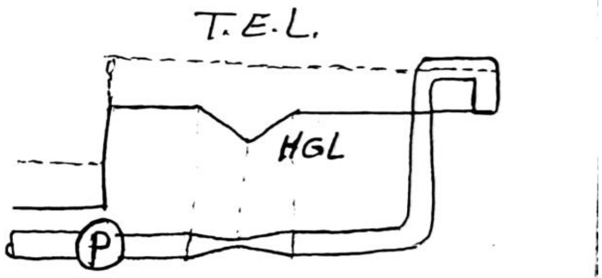
(a) Remove the nozzle.

(b) Replace the nozzle by diffuser.

(c) Remove the nozzle and bend the pipe (7-8) 90° downward.

8 No losses





Problem (9)

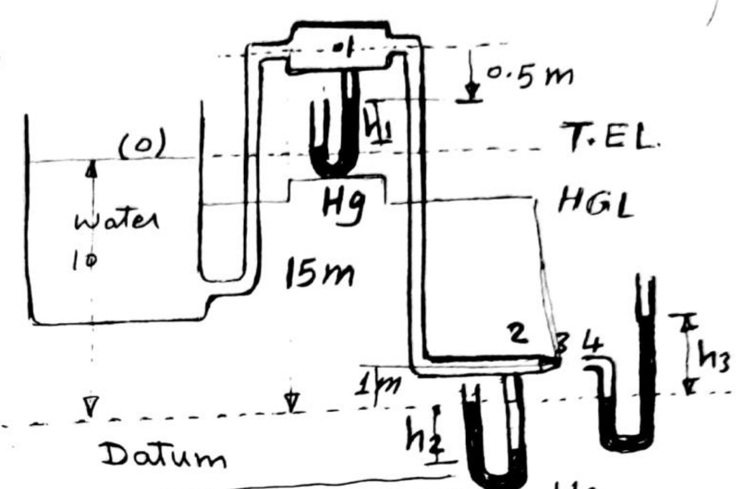
If the water flows steadily through the system shown. Determine the jet velocity, the flow rate and manometric reading **h_1**, **h_2** and **h_3**.

أى نقطة فوق HGL الضغط عندها سالب

(9)

$d_1 = 100$ mm
$d_2 = 50$ mm
$d_3 = 20$ mm (Nozzle)

Req: $V_3 = ?$ $Q = ??$
$h_1 = ?$ $h_2 = ?$ $h_3 = ?$



sol: $\therefore Q = V_1 A_1 = V_2 A_2 = V_3 A_3$

$\therefore V_2 = 0.16\, V_3$
$\therefore V_1 = 0.04\, V_3$

Bernoulli (0) (3)

$$0 + 10 + 0 = 0 + 1 + \frac{V_3^2}{2g}$$

$$\boxed{\begin{aligned} &\therefore V_3 = 13.29 \tfrac{m}{s} \\ &\therefore V_2 = 2.13 \tfrac{m}{s} \\ &\therefore V_1 = 0.53 \tfrac{m}{s} \end{aligned}}$$

$$\therefore Q = V_3 \cdot \frac{\pi}{4} d_3^2 \qquad \boxed{\therefore Q = 4.18 \times 10^{-3} \frac{m^3}{s}}$$

Bernoulli (0)(1)

$$0 + 10 + 0 = \frac{P_1}{\gamma} + 15 + \frac{(0.53)^2}{2g} \qquad \therefore P_1 = -49.19 \text{ KPa}$$

$$\therefore h_1 = 0.33 m$$

Manometer:

$$0 = -49.19 \times 10^3 + (1000)(9.81)(0.5) + (13600)(9.81) h_1$$

Bernoulli (0)(2)

$$0 + 10 + 0 = \frac{P_2}{\gamma} + 1 + \frac{(2.13)^2}{2(9.81)} \qquad \therefore P_2 = 86 \text{ KPa}$$

Manometer:

$$\therefore h_2 = 0.775 m$$

$$(13600)(9.81) h_2 = 86 \times 10^3 + (1000)(9.81)(1) + (1000)(9.81) h_2$$

Bernoulli (0)(4) داخل الماسورة

$$0 + 10 + 0 = \frac{P_4}{\gamma} + 1 + 0 \qquad \therefore P_4 = 88.29 \text{ KPa}$$

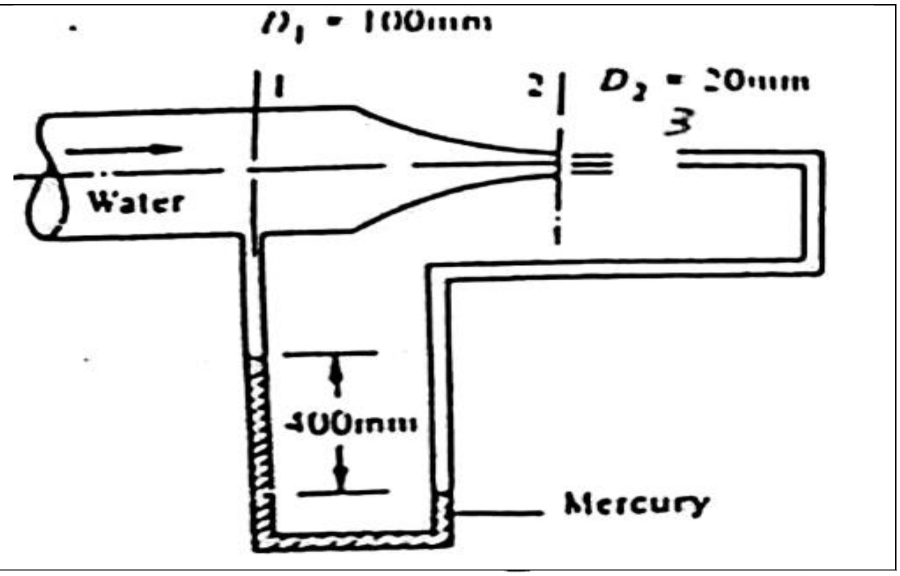
Manometer:

$$\therefore h_3 = 0.735 m$$

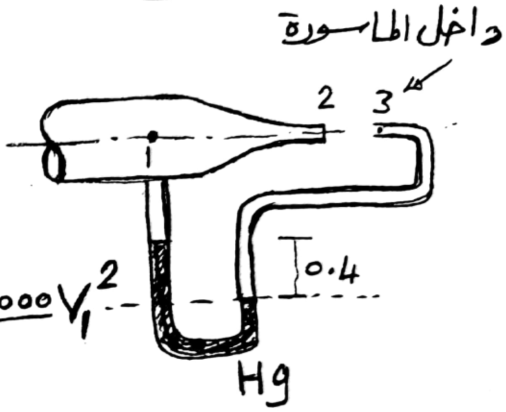
$$88.29 \times 10^3 + (1000)(9.81)(1) = (13600)(9.81) h_3$$

Problem (10)

Calculate the discharge, Q through the nozzle shown in figure.



$\boxed{10}$ $D_1 = 100$ mm $d_2 = 20$ mm



Bernoulli ① ③:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_3}{\gamma} + \overset{0}{\cancel{\frac{V_3^2}{2g}}} + \cancel{Z_3}$$

$$\therefore \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_3}{\gamma} \qquad \therefore P_3 = P_1 + \frac{1000}{2} V_1^2$$

Manometer:

$$P_1 + (0.4)(13600)(9.81) = P_3 + (0.4)(1000)(9.81)$$

$$\cancel{P_1} + 53366.4 = \cancel{P_1} + \frac{1000}{2} V_1^2 + 3924$$

$$\boxed{\therefore V_1 = 9.94 \text{ m/s}} \ \#$$

$$\therefore Q = V_1 \cdot A_1 \qquad \therefore Q = (9.94)\frac{\pi}{4}(0.1)^2$$

$$\boxed{\therefore Q = 0.0781 \text{ m}^3/\text{sec}}$$

Problem (11)

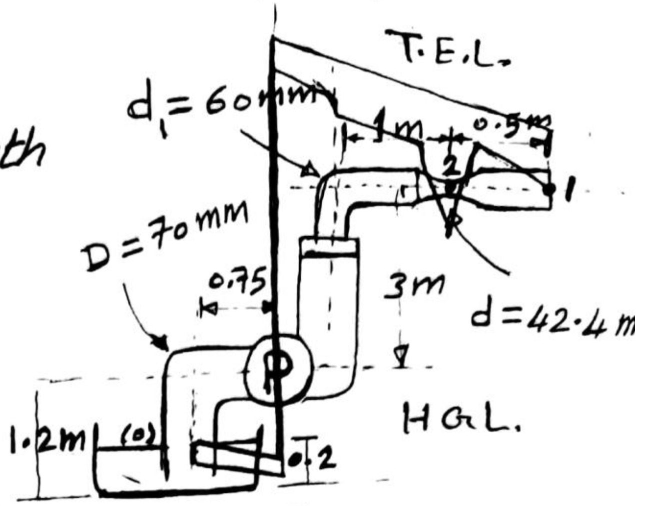
In the system shown, the discharge is 0.018 m^3/sec, energy loss due to friction in the suction and delivery pipes are 0.05m of water per meter length of the pipe. If the pump efficiency is 0.75, determine the power required to drive the pump and sketch the **T.E.L.** and the **H.G.L.**

$\boxed{18}$ $Q = 0.018\ m^3/s$

$h_L = 0.05\ m$ water $/ m$ length

$\eta_p = 0.75$

Power = ??



<u>Sol:</u>

$$h_L = (0.05)[1.2 + 0.75 + 3 + 1 + 0.5]$$
$$= 0.323\ m \text{ water}$$

$$\therefore V_1 = \frac{Q}{\frac{\pi}{4} d_1^2} = \frac{0.018}{\frac{\pi}{4}(0.06)^2} = \boxed{6.37\ m/s}$$

<u>Bernoulli (0)(1)</u>

$$0 + 0 + 0 + H_p = 0 + 4 + \frac{(6.37)^2}{2(9.81)} + 0.323$$

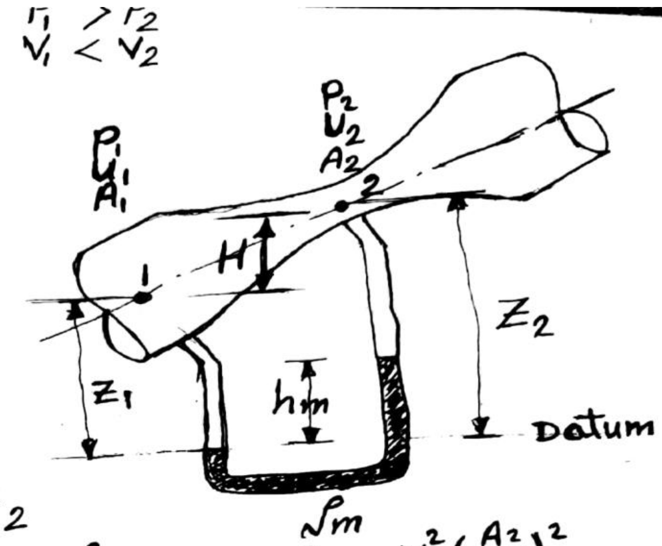
$$\therefore H_p = 6.39\ m \text{ water}$$

$$\therefore \text{Power} = \frac{\rho . g . Q . H_p}{\eta_p}$$
$$= \frac{(1000)(9.81)(0.018)(6.39)}{0.75}$$
$$= 1.504\ KW\ \#$$

غير مهم

Problem (12)

Drive an expression for the flow rate through Venturimeter for steady incompressible flow.



Venturimeter :

① Continuity :

$$V_1 A_1 = V_2 A_2$$

$$V_1 = V_2 \frac{A_2}{A_1} \quad \text{——} \quad ①$$

② Bernoulli ①② :

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1 - P_2}{\gamma} + (Z_1 - Z_2) = \frac{V_2^2 - V_1^2}{2g} = \frac{V_2^2 - V_2^2 (\frac{A_2}{A_1})^2}{2g}$$

$$= \frac{V_2^2}{2g} \left[1 - (\frac{A_2}{A_1})^2\right]$$

$$\frac{P_1 - P_2}{\gamma} + (Z_1 - Z_2) = \frac{V_2^2}{2g A_1^2} (A_1^2 - A_2^2)$$

$$\therefore V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left[\frac{P_1 - P_2}{\gamma} + (Z_1 - Z_2)\right]}$$

$$\because Q = V_2 A_2 \qquad \boxed{\therefore Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g\left[\frac{P_1 - P_2}{\gamma} + (Z_1 - Z_2)\right]}}$$

#

③ Manometer :

$$\because P_1 + \rho g Z_1 = P_2 + \rho g (Z_2 - h_m) + \rho_m \cdot g \cdot h_m$$

$$\therefore (P_1 - P_2) - \rho g (Z_2 - Z_1) = \rho_m \cdot g h_m - \rho g h_m$$

$$\therefore \frac{P_1 - P_2}{\gamma} - (Z_2 - Z_1) = h_m \left(\frac{\rho_m}{\rho} - 1\right)$$

$$\boxed{\therefore Q = \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2g h_m \left(\frac{\rho_m}{\rho} - 1\right)}}$$