

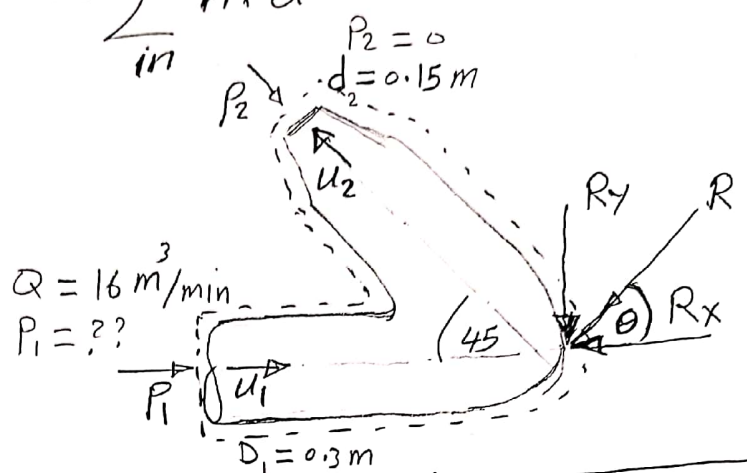
Momentum Equation

$$\sum \vec{F} = \sum_{out} m \cdot \vec{u} - \sum_{in} m \cdot \vec{u}$$

تجميع القوى الخارجية المؤثرة على المائع

wieght + shear pressure + reaction

وزن المائع فقط



Horizontal $\approx w = 0$

$\therefore Q = V_1 A_1 = V_2 A_2$

$\therefore \frac{16}{60} = V_1 \frac{\pi}{4} (0.3)^2 = V_2 \frac{\pi}{4} (0.15)^2$

$\therefore V_1 = 3.77 \text{ m/s}$
 $V_2 = 15.09$

B. equ: $\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$

$\therefore \frac{P_1}{9810} + \frac{(3.77)^2}{2 \cdot 9.81} = \frac{(15.09)^2}{2 \cdot 9.81}$

$\therefore P_1 = 106.75 \text{ KPa}$

Momentum equ: zero

X-D: $P_1 A_1 + P_2 A_2 \cos 45 - R_x = m \cdot (-V_2 \cos 45 - V_1)$

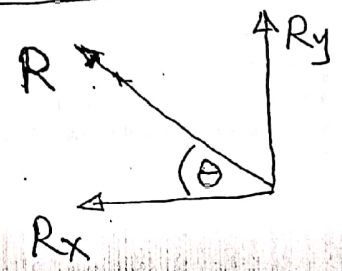
$\therefore (106.75 \times 10^3) \frac{\pi}{4} (0.3)^2 - R_x = 1000 \left(\frac{16}{60} \right) [15.09 \cos 45 + 3.77]$
 $\therefore R_x = 11.396 \text{ KN}$

Y-D: $0 + 0 - R_y = m \cdot (V_2 \sin 45 - 0)$

$\therefore -R_y = 1000 \left(\frac{16}{60} \right) (15.09 \sin 45)$
 $\therefore R_y = -2.845 \text{ KN}$

$\therefore R = \sqrt{R_x^2 + R_y^2} = 11.745 \text{ KN}$

$\therefore \theta = \tan^{-1} \frac{2.845}{11.396} = 14^\circ$



Horizontal

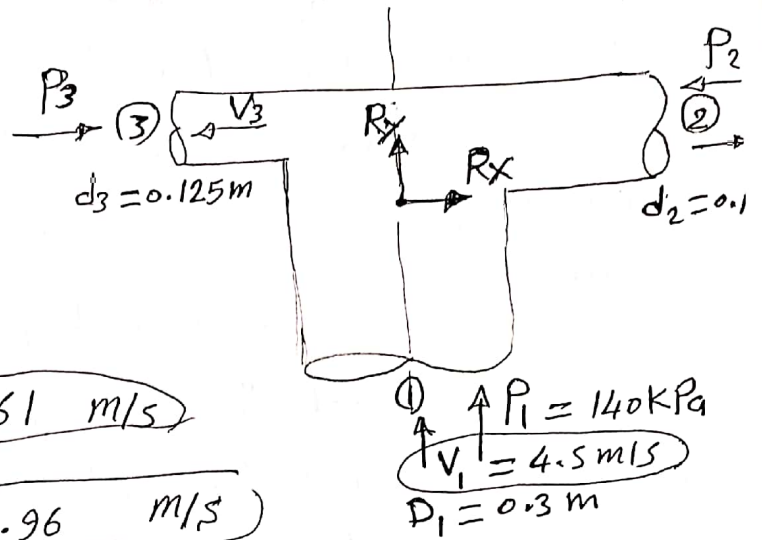
$$Q_2 = Q_3$$

$$\therefore Q_1 = V_1 \frac{\pi}{4} D_1^2 = 0.318 \frac{m^3}{sec}$$

$$\therefore Q_2 = Q_3 = 0.159 \frac{m^3}{sec}$$

$$\therefore V_2 = \frac{0.159}{\frac{\pi}{4} (0.175)^2} = 6.61 \text{ m/s}$$

$$\therefore V_3 = \frac{0.159}{\frac{\pi}{4} (0.125)^2} = 12.96 \text{ m/s}$$



B.eq:

① ②

$$z_1 = z_2 \text{ ??}$$

$$\therefore \frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$$

$$\therefore P_2 = 128.3 \text{ KPa}$$

B.eq:

① ③

$$z_1 = z_3 \text{ ??}$$

$$\therefore \frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_3}{\rho g} + z_3 + \frac{V_3^2}{2g}$$

$$\therefore P_3 = 66.14 \text{ KPa}$$

Momentum:

X-D:

$$P_3 A_3 - P_2 A_2 + R_x = (-m_3 V_3 + m_2 V_2) - (0)$$

$$\therefore (66.14 \times 10^3) \left(\frac{\pi}{4}\right) (0.125)^2 - (128.3 \times 10^3) \left(\frac{\pi}{4}\right) (0.175)^2 + R_x =$$

$$\therefore R_x = 1.265 \text{ KN}$$

$$- (159)(12.96) + (159)(6.61)$$

$$\therefore R_y = -11.327 \text{ KN}$$

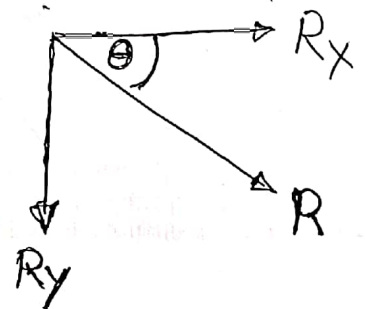
Y-D:

$$\therefore P_1 A_1 + R_y = 0 - m_1 V_1$$

$$\therefore (140 \times 10^3) \frac{\pi}{4} (0.3)^2 + R_y = - (318)(4.5)$$

$$\therefore R = 11.397 \text{ KN}$$

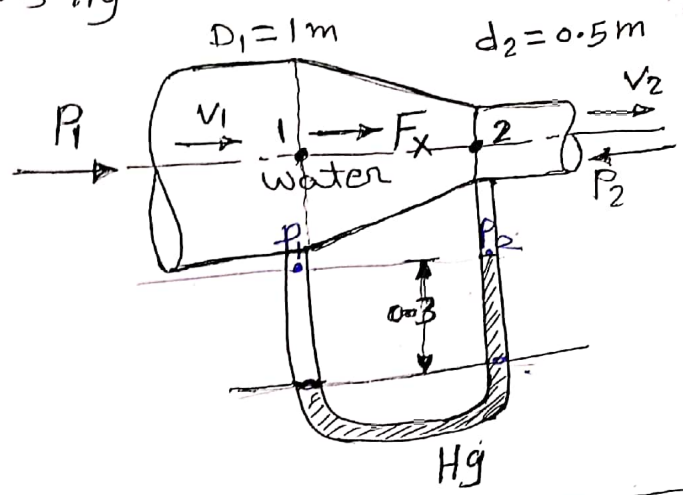
$$\therefore \theta = \tan^{-1} \frac{11.327}{1.265} = 83.6^\circ$$



1 is
rem
MVA

$\circ P_1 = 29430 \text{ Pa}$
 $\frac{P_1}{\rho g} = 3 \text{ m} \quad h_m = 0.3 \text{ Hg}$

Req: $Q = ?$ or $m' = ?$
 $R_x = ?$ T.E.L. H.G.L.



Sol:

$V_1 \frac{\pi}{4} D_1^2 = V_2 \frac{\pi}{4} d_2^2$

$\circ V_2 = 4V_1 \quad \#$

Manometer:

$\frac{29430}{\rho} + 0.3 \times 10^3 \times g = P_2 + 0.3 \times 13600 \times g$

$\circ P_2 = -7652 \text{ Pa}$

B.Eq:

$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$

$\circ 3 + \frac{V_1^2}{2g} = \frac{-7652}{(1000)(9.81)} + \frac{(4V_1)^2}{2g}$

$\circ V_1 = 2.22 \text{ m/s}$

$\circ V_2 = 8.88 \text{ m/s}$

$\circ Q = V_1 A_1 = (2.22) \frac{\pi}{4} (1)^2$

$\circ Q = 1.744 \text{ m}^3/\text{sec}$

Momentum:

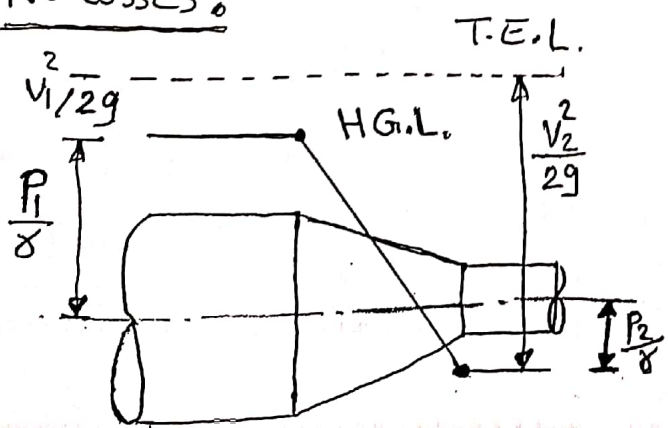
$\circ \sum F = m'(V_{x0} - V_{xi})$

$\circ P_1 A_1 - P_2 A_2 + F_x = m'(V_2 - V_1)$

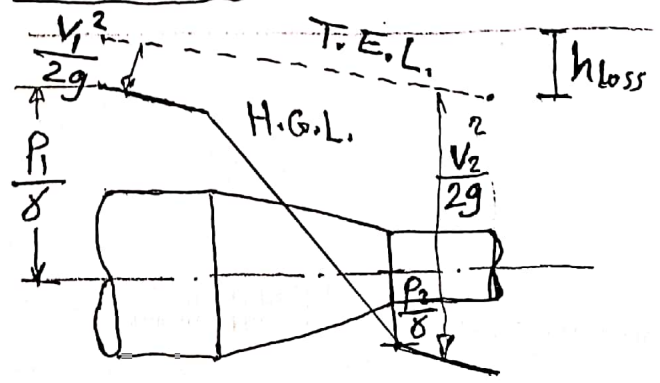
$\circ 29430 \left(\frac{\pi}{4}\right) (1)^2 + 7652 \left(\frac{\pi}{4}\right) (0.5)^2 + F_x = (1744)(8.88 - 2.22)$

$\circ F_x = -13.002 \text{ KN}$

No losses:

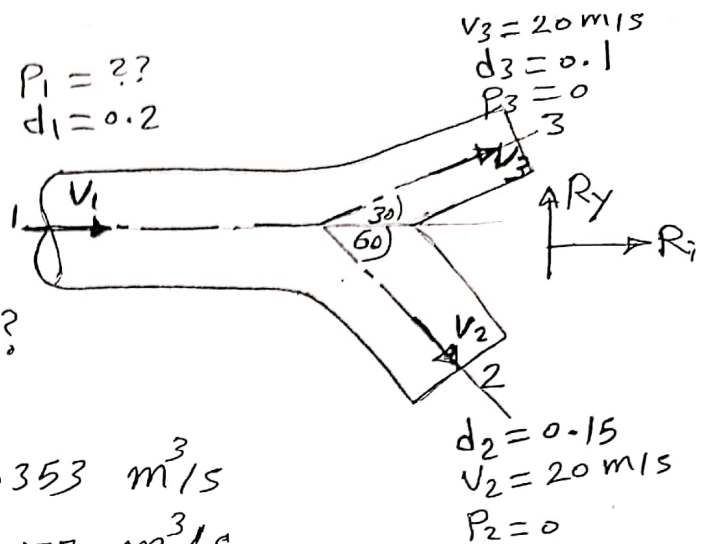


with losses:



Horizontal
 $Z_1 = Z_2 = Z_3$
 No losses

$Q_1 = ??$ $R_x = ?$ $R_y = ?$



Sol^o

$Q_2 = V_2 \frac{\pi}{4} d_2^2 = 20 \frac{\pi}{4} (0.15)^2 = 0.353 \text{ m}^3/\text{s}$

$Q_3 = V_3 \frac{\pi}{4} d_3^2 = 20 \frac{\pi}{4} (0.1)^2 = 0.157 \text{ m}^3/\text{s}$

$Q_1 = Q_2 + Q_3 = 0.510 \text{ m}^3/\text{s}$

$V_1 = \frac{Q_1}{\frac{\pi}{4} d_1^2}$

$V_1 = 16.25 \text{ m/s}$

$d_2 = 0.15$
 $V_2 = 20 \text{ m/s}$
 $P_2 = 0$

$V_3 = 20 \text{ m/s}$
 $d_3 = 0.1$
 $P_3 = 0$

B. eq^y: $\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$

$P_1 = 67969 \text{ Pa}$

Momentum:

X-D: $P_1 A_1 - 0 - 0 + R_x = m_2 V_2 \cos 60 + m_3 V_3 \cos 30 - m_1 V_1$

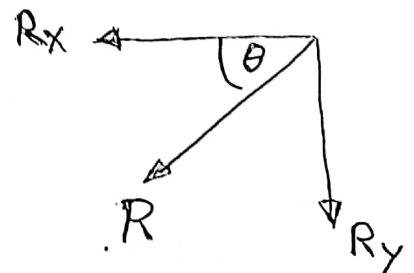
$R_x = -4.173 \text{ kN}$

Y-D: $R_y = (m_3 V_3 \sin 30 - m_2 V_2 \sin 60) - (0)$

$R_y = -4.544 \text{ kN}$

$R = \sqrt{(4173)^2 + (4544)^2}$
 $= 6.169 \text{ kN}$

$\theta = \tan^{-1} \frac{4544}{4173}$
 $= 47.4^\circ$



Vertical $z_1 \neq z_2$

$$D_1 = D_2 = 0.2 \text{ m}$$

weight of water = 250 N

(ρ of bend = 1000 N) غير موثر

Req: $Q = ??$ $R_x = ?$ $R_y = ?$

Sol:

T.E.L. H.G.L.

B. equ: $\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$

$$\therefore P_1 = 9810 \text{ Pa}$$

$$\therefore V_1 = 7.67 \text{ m/s}$$

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_3}{\rho g} + z_3 + \frac{V_3^2}{2g}$$

Momentum equ:

$$Q = V_1 \cdot A_1 = (7.67) \frac{\pi}{4} (0.2)^2$$

$$\therefore Q = 0.241 \text{ m}^3/\text{se}$$

X-D: $\rho P_1 A_1 - R_x = m'(0 - V_1)$

$$\therefore R_x = 2156.7 \text{ N}$$

Y-D: $R_y - W = m'(V_2 - 0)$

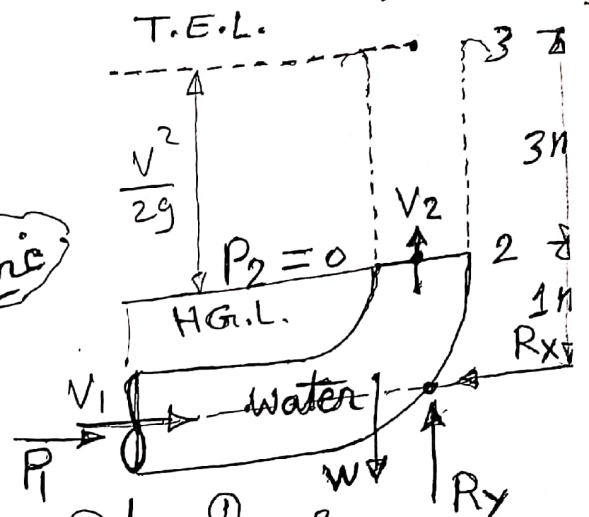
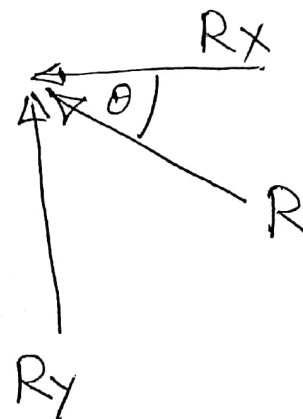
$$\therefore R_y = 2098.5 \text{ N}$$

$$\therefore R = \sqrt{(2156.7)^2 + (2098.5)^2}$$

$$= 3009.1 \text{ N}$$

$$\therefore \theta = \tan^{-1} \frac{2098.5}{2156.7}$$

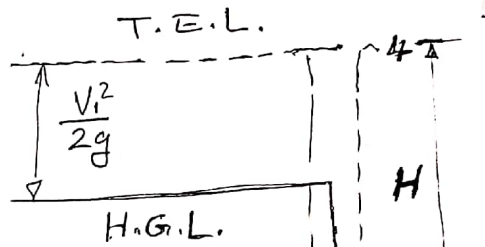
$$= 44.2^\circ$$



$$Q = 0.241 \text{ m}^3/\text{s} \quad V_4 = 0$$

$$d_3 = 0.1 \text{ m} \quad D_1 = 0.2 \text{ m}$$

T.E.L. H.G.L. $P_1 = ??$
 $H = ??$ $R_x = ?$ $R_y = ?$



Sol^o

$$V_1 = 7.67 \text{ m/s}$$

$$V_3 = \frac{0.241}{\frac{\pi}{4} (0.1)^2} = 30.7 \text{ m/s}$$

B. equ^o

$$\textcircled{3} \textcircled{4} \quad \frac{P_3}{\rho g} + z_3 + \frac{V_3^2}{2g} = \frac{P_4}{\rho g} + z_4 + \frac{V_4^2}{2g}$$

$$\therefore H = 48.04 \text{ m}$$

B. equ^o

$$\textcircled{1} \textcircled{4} \quad \frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_4}{\rho g} + z_4 + \frac{V_4^2}{2g}$$

$$\therefore P_1 = 451.7 \text{ kPa}$$

Momentum^o

X-D^o

$$P_1 A_1 - R_x = m'(0 - V_1)$$

$$\therefore R_x = 16.04 \text{ kN}$$

$$(451.7 \times 10^3) \frac{\pi}{4} (0.2)^2 - R_x = 241(-7.67) \quad \text{ff}$$

Y-D^o

$$R_y - W = m'(V_3 - 0)$$

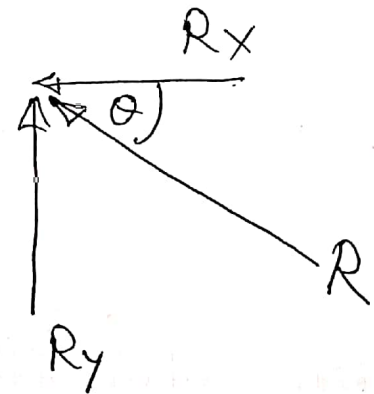
$$\therefore R_y = 8.65 \text{ kN}$$

$$\therefore R = \sqrt{(16.04)^2 + (8.65)^2}$$

$$= 18.22 \text{ kN}$$

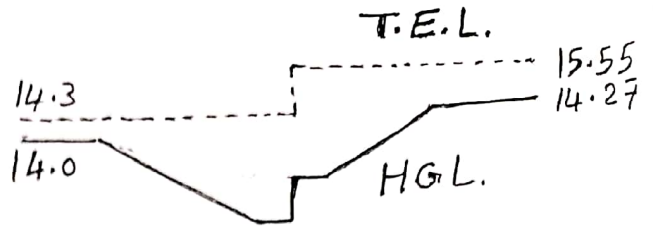
$$\theta = \tan^{-1} \frac{8.65}{16.04}$$

$$= 28.3^\circ$$



turbine power = 12000 W
No losses

$$\eta_T = 100\%$$



Req:

$$P_2 = ?? \quad F_x = ??$$

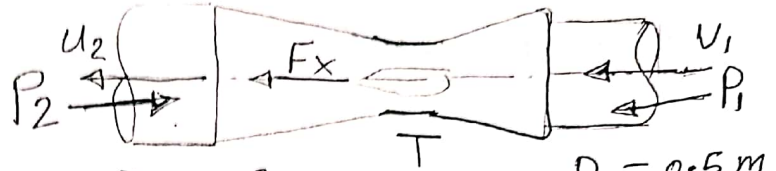
Sol:

$$Q_1 = (5) \frac{\pi}{4} (0.5)^2$$

$$Q = 0.982 \text{ m}^3/\text{sec}$$

$$\therefore U_2 = \frac{0.982}{\frac{\pi}{4} (0.7)^2}$$

$$\therefore U_2 = 2.55 \text{ m/s}$$



$$D_2 = 0.7 \text{ m}$$

$$U_2 = ?$$

$$P_2 = ?$$

$$D_1 = 0.5 \text{ m}$$

$$U_1 = 5 \text{ m/s}$$

$$P_1 = 140 \text{ kPa}$$

$$\therefore P = \rho \cdot g \cdot Q \cdot H_T$$

$$\therefore H_T = \frac{12000}{(10^3)(9.81)(0.982)} = 1.246 \text{ m}$$

B. equ: $\frac{P_1}{\rho g} + \frac{U_1^2}{2g} - H_T = \frac{P_2}{\rho g} + \frac{U_2^2}{2g}$

$$\therefore \frac{140 \times 10^3}{(10^3)(9.81)} + \frac{5^2}{2 \cdot 9.81} - 1.246 = \frac{P_2}{\rho g} + \frac{(2.55)^2}{2 \cdot 9.81}$$

$$\therefore P_2 = 137.03 \text{ kPa}$$

Momentum:

$$\therefore P_1 A_1 - P_2 A_2 + F_x = \dot{m} (U_2 - U_1)$$

$$\therefore (140 \times 10^3) \frac{\pi}{4} (0.5)^2 - (137 \times 10^3) \frac{\pi}{4} (0.7)^2 + F_x = 982 (2.55 - 5)$$

$$\therefore F_x = 22.83 \text{ kN}$$

$$C_d = 0.97$$

$$m = 5 \text{ kg}$$

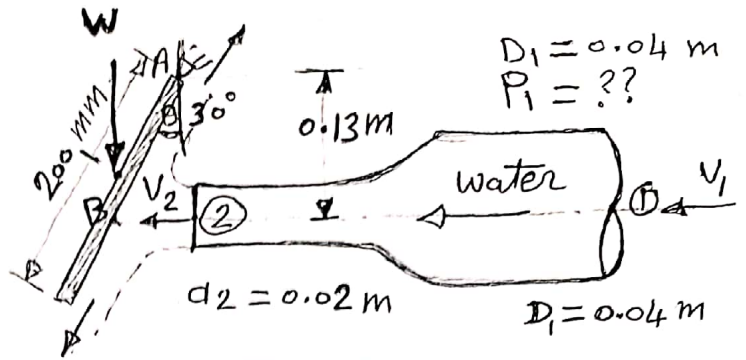
$$\theta = 30^\circ$$

$$P_2 = 0$$

$$d_2 = 0.02 \text{ m}$$

$$D_1 = 0.04 \text{ m}$$

$$P_1 = ??$$



Sol: $m' = C_d \cdot \rho \cdot V_2 \cdot A_2$
Momentum:
 $\sum F = m'(V_o - V_i)$

$$-F_w = m'(0 - V_2)$$

$$F_w = C_d \cdot \rho \cdot A_2 \cdot V_2^2$$

$$F_w = 0.3047 V_2^2 = F_w$$

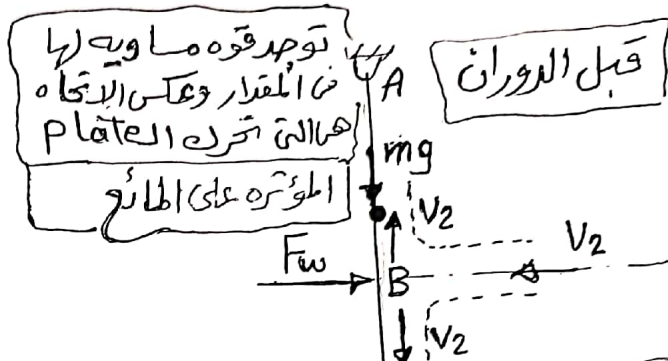


Plate equilibrium:

$$\sum M_A = 0$$

$$F_w (0.13) = mg (0.1 \sin 30)$$

$$0.3047 V_2^2 (0.13) = (5)(9.81)(0.05)$$

$$V_2 = 7.87 \text{ m/s}$$

Continuity: $V_2 \frac{\pi}{4} (0.02)^2 = V_1 \cdot \frac{\pi}{4} (0.04)^2$

$$V_1 = 1.97 \text{ m/s}$$

B. eqn: $\frac{P_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + z_2 + \frac{V_2^2}{2g}$

$$P_1 = 29028 \text{ Pa}$$

$$= 29.03 \text{ kPa}$$

Hinge Reactions: غير مطلوب

$$\sum F_x = 0 \implies R_H = F_w = 18.87 \text{ N}$$

$$\sum F_y = 0 \implies R_V = mg = 49.05 \text{ N}$$

$$R = \sqrt{R_V^2 + R_H^2} = 52.6 \text{ N}$$

$$\theta = \tan^{-1} \frac{49.05}{18.87} = 79^\circ$$

8] $\rho \tilde{A} U \Delta$

Sol:

Momentum in N-D:

$$\Sigma F = m'(V_o - V_i)$$

$$\circ \circ - F_w \cos 30 =$$

$$Cd \cdot \rho \cdot A_2 V_2 (0 - V_2 \cos 30)$$

$$\circ \circ F_w = 0.3047 V_2^2 \quad \text{--- (1)}$$

Cot. equ: $V_1 = V_2 \left(\frac{0.02}{0.04} \right)^2 \rightarrow V_1 = 0.25 V_2$

B. equ: $\frac{P_1}{\rho g} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2^2}{2g}$

$$\circ \circ P_1 = 468.75 V_2^2$$

Plate equilibrium:

$$\Sigma M_A = 0 \rightarrow \circ \circ F_w (0.13) = mg (0.05)$$

$$\circ \circ P_1 = 29028 \text{ Pa} \quad \#$$

$$\circ \circ F_w = 0.3047 V_2^2$$

$$\circ \circ V_2 = 7.87 \text{ m/s}$$

$$\circ \circ V_1 = 1.97 \text{ m/s}$$

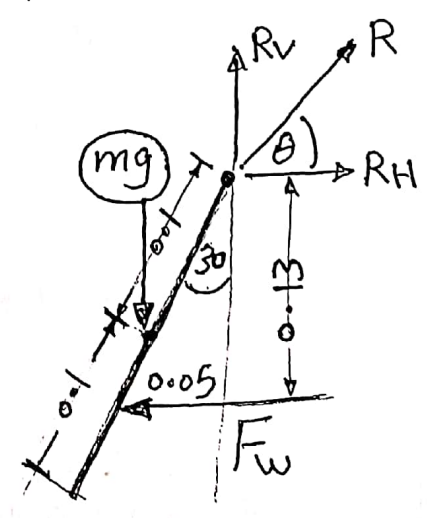
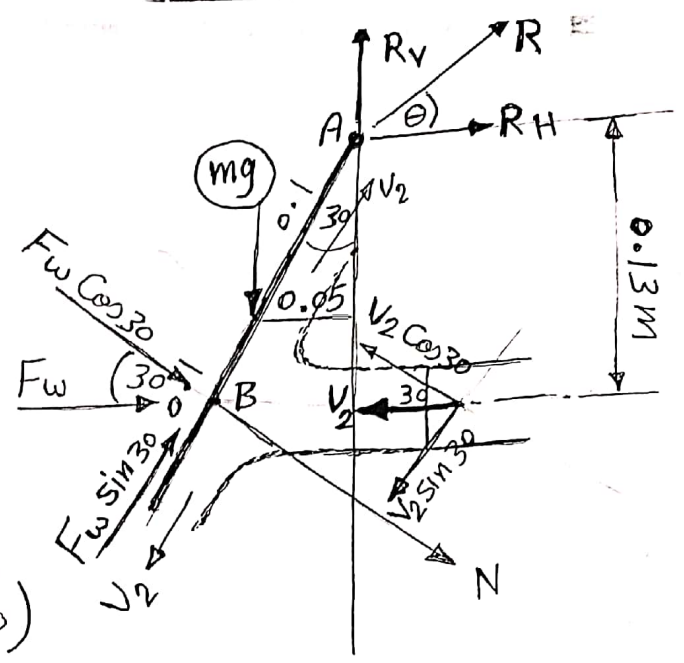
Hinge Reaction:

$$R_v = mg = 49.05 \text{ N}$$

$$R_H = F_w = 18.87 \text{ N}$$

$$\circ \circ R = 52.55 \text{ N} \quad \#$$

$$\circ \circ \theta = 79^\circ \quad \#$$



$V_1 = 36 \text{ m/s}$
 $U = 15 \text{ m/s}$
 $d_{\text{jet}} = 0.1 \text{ m}$

$V_{r2} = 0.85 V_{r1}$

$\beta_1 = ?$ $\beta_2 = ?$ $F_x = ?$

Power = ? $\eta = ?$

Sol^o
 $\tan \beta_1 = \frac{V_1 \sin 30}{V_1 \cos 30 - U}$

$V_{r1}^2 = (V_1 \sin 30)^2 + (V_1 \cos 30 - U)^2$

$V_{r2} = 0.85 (24.2)$

$V_2^2 = (20.57)^2 - (15)^2$

$\tan \beta_2 = \frac{14.08}{15}$

Continuity: $m' = \rho \cdot V_1 \cdot \frac{\pi}{4} d_j^2 = 10^3 (36) \frac{\pi}{4} (0.1)^2$

$m' = 282.7 \text{ kg/s}$

Momentum equ: X-D: V_r (تجاه \vec{u})

$\sum F_x = m' (V_{x0} - V_{xi})$

$F_x = (282.7) (-V_{r2} \cos \beta_2 - V_{r1} \cos \beta_1)$

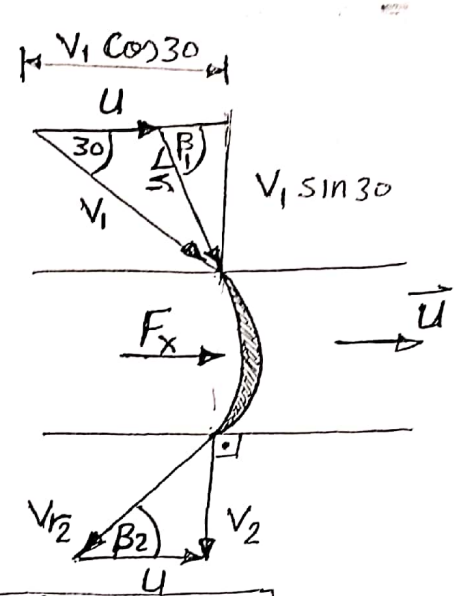
$F_x = -8.817 \text{ KN}$

$\text{Power} = F_x \cdot U$

$= (8.817)(15) = 132.25 \text{ KW}$

$\eta = \frac{\text{out put } W}{\text{in put } W} = \frac{132.25 \times 10^3}{\frac{1}{2} (282.7) (36)^2}$

$\eta = 72.2 \%$



$\beta_1 = 48^\circ$

$V_{r1} = 24.2 \text{ m/s}$

$V_{r2} = 20.57 \text{ m/s}$

$V_2 = 14.08 \text{ m/s}$

$\beta_2 = 43.2^\circ$